Units of Measure and Scaling Factors

UoM-TN-12

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1 Introduction

In this short paper the aim is to introduce and argue for the appropriate usage of units of measure (UoM), combined with the relevant application of scaling factors and how values with their UoM should be displayed. Finally, a consideration is given to both the *accuracy* and *precision* of values.

The information contained in this paper can be obtained from multiple on-line resources, and further details can be found within the material produced by the Bureau International des Poids et Mesures (BIPM) [3], National Physical Laboratory (NPL) [6] or with National Institute of Standards and Technology (NIST) [5].

2 Lessons from History

Why is the proper usage and portrayal of UoM important to consider? Two examples should help to illustrate the point.

As a student engineer, gaining an appreciation of the work in a machine shop, two sets of machines were in operation; imperial and SI both colour coded and labelled. In making a cut on an SI machine the setting was specified as 1 mm (0.001 m). However, an apprentice, working on an imperial machine set the cut to 1″, approximately 25 times larger. When milling tool and steel met one had to fail, the milling tool shattered firing metal fragments across the machine shop. Fortunately, no one was hurt.

A more public and expensive example of the use of the incorrect UoM can be seen in one of NASA's space programmes, the unmanned Mars Climate Orbiter (MCO). The report into the loss of the MCO [4] identified, through root cause analysis, that cause was due to

> "the failure to use metric units in the coding of a ground software file,"Small Forces," used in trajectory models. Specifically, thruster performance data in English units instead of metric units was used",

that is, the incorrect UoM in a software module caused the MCO mission to fail.

Thus the benefit for the use and exchange of information using the appropriate UoM is the correct understanding of quantities and measures being employed, reduced misunderstanding and less loss of resources.

3 Systems for Units of Measure

There have been many systems defined for the measurement of units including:

- Imperial;
- American;
- CGS (centimetre, gram, second);
- MKS (metre, kilogram, second);
- SI (International System of Units).

3.1 SI Units

In general for science and engineering the SI units scheme has been widely accepted and adopted [3]. The SI units have a number of fundamental units from which other units are derived.

The seven primary, fundamental, or base SI units are:

- kilogram (kg) for mass;
- metre (m) for length;
- second (s) for time;
- ampere (A) for electric current;
- mole (mol) for amounts;

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- candela (cd) for luminous intensity;
- kelvin (K) for temperature.

Other units within the SI system are derived from these seven, §3.2.

3.2 Derived Units

Derived units are constructed from primary units, or other derived units, and provide a convenient short form method for expressing quantities. Ultimately, derived units can be decomposed into primary units, but this is rarely performed. The development of derived units can be illustrated by considering the SI derived units for force, energy (or work) and power.

Force is measured in newtons (N) and is defined as that required to accelerate a one kilogram mass at a rate of one metre per second squared,

$$\mathrm{N} = \mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s}^{-2} = \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^2}$$

Energy (work) is measured in joules (J) and is defined as applying a force of one newton through a distance of one metre,

$$\mathrm{J} = \mathrm{N} \cdot \mathrm{m} = \mathrm{kg} \cdot \mathrm{m}^2 \cdot \mathrm{s}^{-2} = \frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^2}$$

Finally, power (the rate at which work is done) is measured in watts (W) and is defined as the rate of energy conversion, that is, joules per second,

$$\mathrm{W} = \mathrm{J} \cdot \mathrm{s}^{-1} = \mathrm{N} \cdot \mathrm{m} \cdot \mathrm{s}^{-1} = \mathrm{kg} \cdot \mathrm{m}^2 \cdot \mathrm{s}^{-3} = \frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^3}$$

The use of derived units reduces the complexity of the UoM to be expressed.

3.3 SI Naming Conventions

It will be noted from §3.1 and §3.2 that for the SI units, whether primary or derived, some unit abbreviations are upper-case while others are lower-case. The rule is that if a unit is named after an individual then it is represented as an upper-case abbreviation symbol. This is illustrated with a few examples in Table 1.

Table 1: Examples of Units Named after Individuals

UoM	Unit Name	Individual
Capacitance	Farad (F)	Michael Faraday
Electric Current	Ampere[1] (A)	André-Marie Ampère
Force	Newton (N)	Sir Isaac Newton
Inductance	Henry (H)	Joseph Henry
Magnetic Flux	Weber (Wb)	Wilhelm Eduard Weber
Temperature	Kelvin (K)	Lord Kelvin

4 Systems of Scaling Factors

Documenting very large or very small values can become difficult where either a large number of trailing or preceding zeros may be required. A system of scaling factors has been prescribed to assist in documenting large and small values. The decimal SI prefixes are presented in Table 2.

Table 2: SI Prefixes

Prefix	Symbol	Scaling
yocto	y	10^{-24}
zepto	z	10^{-21}
atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}
peta	P	10^{15}
exa	E	10^{18}
zetta	Z	10^{21}
yotta	Y	10^{24}

In the digital domain, that is for computers, many of the scalings are based on factors of base 2, binary. There is an evolving strategy for prefixes based upon the use of exponents of 2 as illustrated in Table 3 [2].

Table 3: Binary Prefixes

Prefix	Symbol	Scaling
kibi	Ki[2]	2^{10}
mebi	Mi	2^{20}
gibi	Gi	2^{30}
tebi	Ti	2^{40}
pebi	Pi	2^{50}
exbi	Ei	2^{60}
zebi	Zi	2^{70}
yobi	Yi	2^{80}

Beware, while kilo (10^3) and kibi (2^{10}) are reasonably close in value at 1000 and 1024 respectively, the difference grows significantly when comparing the mega (10^6) and mebi (2^{20}) scales and beyond.

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[1] Frequently abbreviated to Amp.

[2] Note that kibi (Ki) uses an upper-case K, kilo (k) uses a lower-case k.

5 Presentation of Quantities

In presenting numbers in text assorted style guides, for example Shipley's [7], layout many rules for the presentation of numerical quantities. A basic set of rules is as follows.

The decimal separator is a period ".", while digits are presented in groups of three. The usual style is to space separate the groups but in some fields comma separation is used. A space should also be left between the digits and the UoM.

Examples of the two approaches are

55 234.012 372 kg

or

21,987.154,21 m.

For large or small quantities scaling factor prefixes or engineering (scientific) notation can be used. When using scaling factor prefixes the use of spaces or commas to group numbers into groups of three is maintained.

For example,

1,055.245 μA $\equiv$ 1.055,245 mA.

For engineering notation the quantity is expressed as a factor of 10, for example,

1.055,245 $\times 10^{-3}$ A.

Scaling factors and engineering notation should both be used such that the quantity presented is between 0.1 and 1000.

6 Accuracy versus Precision

Accuracy and precision are defined as follows [1]:

Precision

1. the quality of being precise;
2. characterized by or having a high degree of exactness.

It is the latter definition we are concerned about in this discussion.

Accuracy

1. faithful measurement or representation of the truth; correctness; precision;
2. the degree of agreement between a measured value and the standard or accepted value for that measurement.

These two properties of precision and accuracy can be illustrated by the use of the value π in admittedly some contrived examples. π is frequently simply expressed as 3.14 but a more precise expression is 3.141 592. π could be expressed as 3.14 which is more accurate than expressing it as 3.15.

Note, if a quantity is expressed precisely it does not necessarily mean that it is accurate. For example, expressing π as 3.287 162 is more precise than expressing π as 3.14 but is significantly less accurate.

7 Summary

In this short article the aim has been to present the appropriate use of UoM and scaling factors. It has extended into the representation of quantities and to a simple discussion of accuracy and precision.

References

[1] *Collins English Dictionary.* HarperCollins Publishers, third edition, 1991. ISBN 0-00-433286-5.

[2] International Electrotechnical Commission. *Letter symbols to be used in electrical technology - Part 2: Telecommunications and electronics.* 3 edition, 2005. IEC60027-2.

[3] Bureau International des Poids et Mesures. *Le Systme international dunits (SI).* 8 edition, 2006. ISBN 92-822-2213-6.

[4] NASA. Mars Climate Orbiter Mishap Investigation Board Phase I Report. Technical report, NASA, November 1999.

[5] National Institute of Standards and Technology. The NIST Reference on Constants, Units, and Uncertainty.

[6] National Physical Laboratory. Measurement Units.

[7] Lawrence H. Freeman Terry Bacon. *Style Guide.* Shipley Associates, 1990. ISBN 0-933427-00-X.

Abbreviations

BIPM	Bureau International des Poids et Mesures (International Bureau of Weights and Measures)
CGS	centimetre, gram, second
IEC	International Electrotechnical Commission
MCO	Mars Climate Orbiter
MKS	metre, kilogram, second
NASA	National Aeronautics and Space Administration
NIST	National Institute of Standards and Technology
NPL	National Physical Laboratory
SI	Système International d´Unités
UoM	Unit of Measure